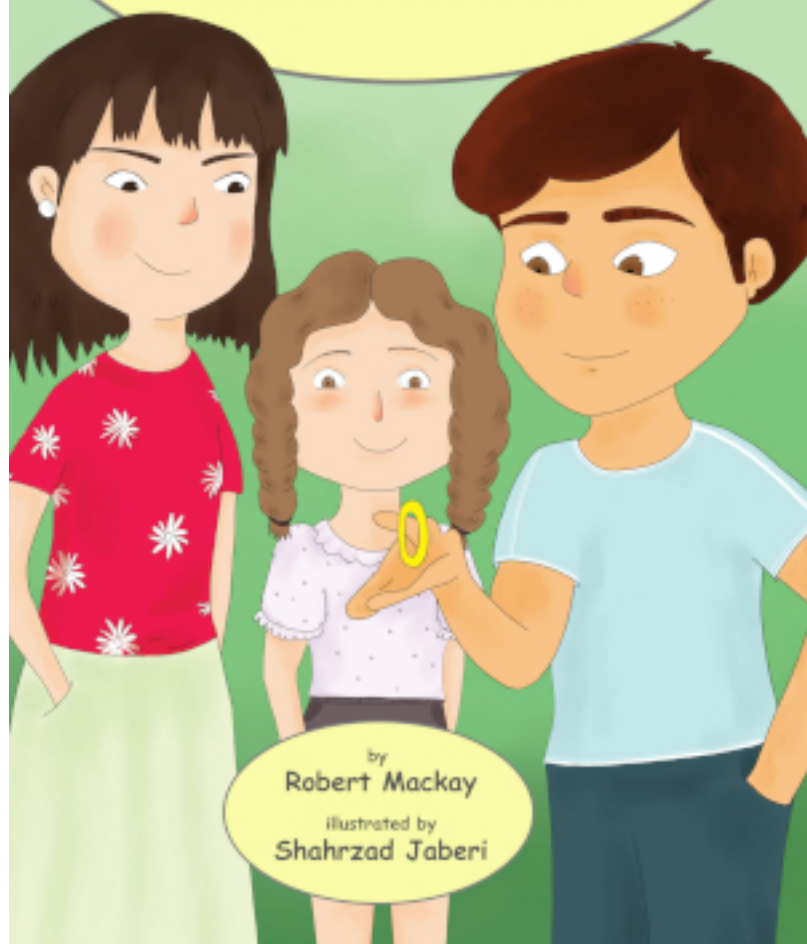


YOU MAKE MATH

A math book for people who hate math
and for those who love it!



by
Robert Mackay

illustrated by
Shahrazad Jafari

You Make Math: A math book for people
who hate math, and for those who love it!

Robert Mackay

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Dedication

For the playful and curious at heart.

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1. Toby Meets Zero



Brrring! The bell rang. Children started pouring out of the school. Laughing, shouting, and playing, some climbed onto buses, others met their parents, and the rest started running home. After most of the children had gone, one sad-faced boy named Toby walked slowly out of the building.

“Oh no, what am I going to do?” he said to himself. “Tomorrow is the math test and I can’t do it. I hate math.” He walked out of the school yard and down the sidewalk. Talking to himself, he started getting angry.

"I hate math," he repeated. "Who ever invented math anyway? Math is just one big ZERO!"

"Why do you say that?" asked a voice.

Toby looked around. There was nobody near him. The other children were all gone. "Who said that?" he asked.

"I did," said the voice again.

Toby looked down. There by the edge of a sidewalk was a bright, shiny object. He reached down and picked it up. It was a large golden ring.

"What a nice ring," he said.

"I'm not a ring," it said. "I'm a zero. My name is Zero."

"Huh," said Toby, "a talking zero. Where did you come from?"

"You created me, Toby," said Zero.

"Me?" asked Toby in astonishment. "How?"

"I'm a product of your imagination. When you were coming out of the school building just now, you said that math is just one big ZERO. Ta-Da! Here I am!"

"Oh, math! Yuck! I can't stand math," said Toby. "I get all mixed up with all those numbers and counting and adding and subtracting and all that stuff."

"Maybe that's because you don't realize that you make math," said Zero.

"I make math? That's a laugh. I don't even understand math," said Toby, turning the ring around in his hand.

He still couldn't believe he was talking to a ring. "How do I make math?"

"Just like you made me," said Zero. "Math is a product of your mind. Math describes how your mind works. Math is the sublime expression of your deepest—"

"Are you crazy?" broke in Toby. "Math is something in a book that gets me all confused. My mind doesn't work like that at all. My mind gets mixed up by math."

"Maybe that's because you don't understand it yet. Would you like to find out a few secrets?"

"Maybe I'm the one who's crazy," thought Toby. "Here I am talking to a little golden ring about math and—"

"No, Toby, you're not crazy," said Zero.

"Wow, you can read my mind!" cried Toby. "This is too much."

"Relax," said Zero. "After all, I'm a product of your mind. You created me with your mind, so naturally I should be able to read your mind. So, what do you say? Do you want me to help you with math or not?"

"Sure, why not?" said Toby. "What do I have to lose? Maybe you can give me the answers to my math test tomorrow."

"I can do better than that," said Zero. "I'll show you how you can pass the test all by yourself, because you make math."

“Yeah, right,” said Toby with a smile. “I make math.” He was quite sure he would fail the test. But it was worth a try. “OK,” he agreed. “You can teach me about math, Zero.” He put Zero in his pocket and walked home, his mind full of questions.

2. Grouping: Actions Make Objects



When Toby got home he went into the kitchen where his mother was preparing dinner. She was sitting at the table, shelling peas. Toby sat down to help. One by one, they picked up pea pods and opened them. Inside each pod there were six or seven bright green peas. They took the peas out of the pod and put them into a bowl. When they were finished, Toby's mother thanked him and he ran outside to play.

"Isn't it beautiful how nature is so organized?" said Zero.

“Oh,” said Toby. He had forgotten all about Zero. He took him out of his pocket. “What do you mean, organized?” he asked Zero.

“Those peas,” said Zero. Look how beautifully they are packaged. Instead of sticking peas here and there all over the pea plant, they are grouped together and held in a single pod.”

Toby laughed, thinking about a pea plant with peas stuck all over it.

“In the pod the peas are protected, and what’s more, when someone wants to pick the peas, they only have to pick the pod, and all the peas come with it.”

“Hmm,” said Toby. “I never looked at it that way before. I only thought about how much work it is to get them out of the pod.”

“Nature organizes things in groups,” said Zero. “This is one of the basic principles of mathematics. And people organize things in groups, too. It is a natural way to make things easy and simple.”

“What do you mean?” asked Toby.

“Well, what did you and your mother do with the peas?” asked Zero.

“We put them in a bowl,” said Toby.

“Why?” asked Zero.

“Because if we didn’t they would roll all over the table, spill on the floor, and go off in all directions.”

“Right,” said Zero. “And in the bowl, you can carry them around the kitchen.”

“Of course,” said Toby. “But what does that have to do with math?”

“Well,” said Zero, “math creates and describes groups.”

“How so?” asked Toby.

“Think about it,” said Zero. “What is a group?”

“A group is many things put together,” answered Toby.

“Correct,” said Zero. Toby smiled. This was easy.

“Now,” said Zero, “Who puts the group together?”

Toby thought about the peas. “The pea plant put them together in the pod, and Mother and I put them together in the bowl.”

“Correct again,” said Zero. “Nature organizes things in groups, like the pea plant. And you yourself organize things in groups, too. It makes them easier to handle.”

“Sure,” said Toby. “I put my pencils in my pencil box, I put my marbles in a bag, I keep all my socks together in a drawer. It is easier to carry things around in a bag or a bowl. But what does this have to do with math?”

“Well,” said Zero, “here's something that most people don't realize. In math, actions can become objects.”

“Actions can become objects,” repeated Toby. “What does that mean?”

“It means that sometimes you think of actions as things. Like when you see kids playing soccer, you say it's

a soccer game. The action of playing is called a “game”, and so it becomes a thing. Or when cars are racing, we call it a race.”

“I see,” said Toby. “Like now, we are talking, and it’s a thing. It’s a conversation, right?”

“Right, you got it!”

“So how does that work in math?”

“When you group something, you make a group,” said Zero. “Putting peas in a bowl gives you a bowl of peas. Grouping makes groups.”

“OK,” said Toby.

“And,” continued Zero, “you can create groups in your mind. You can imagine the houses on your street as a group, right? or the countries of the world, or even the stars in the sky. Even though you can’t put them together in a box or bag, you can imagine them as a group, can’t you?”

“Yes,” said Toby. “I can do that.”

“Whenever you group things together in your mind,” said Zero, “your mental action makes a group—a thing. Your action makes an object.”

“And is that math?”

“Part of it,” Zero replied. “Math turns actions into objects all the time. You do it when you make math, but it’s so easy you don’t realize it.”

“Well,” said Toby, “it’s not hard to think of a group.”

2. Grouping: Actions Make Objects

3. All Groups have a Number



“Hi Toby, who are you talking to?” It was Rita, Toby’s friend from next door. She had come over to play.

“Hi Rita. You’ll never guess what happened to me today. I created a talking zero.”

“A what?” asked Rita.

“Here, look.” Toby held out his hand and showed Rita.

“Oh, how cute!” exclaimed Rita.

“Look,” said Toby. “Zero’s turning pink. I think he’s blushing.”

“No I’m not,” said Zero.

“And he talks!” cried Rita. “How wonderful!” Zero turned a deep shade of red.

“He’s teaching me about math,” said Toby.

“You, math?” said Rita. “That’s even more amazing. I didn’t think anyone could teach you about math.”

“Zero has been telling me that I make math. Can you believe that? So far I’ve found out that when we group things together, we are doing math.”

“What do you mean?” asked Rita.

“Tell her, Toby,” said Zero.

“Well,” said Toby, “you make groups of things by putting them together. You can do that with your hands, like putting peas in a bowl, or you can make a group of things with your mind, like thinking of all the white cars in our town.”

“OK,” said Rita. “I’m thinking of all the white cars in our town. What does that have to do with math?”

“Here’s where it gets interesting,” said Zero. “Think, now. What is one thing that every group has in common. What does every group have?”

Toby and Rita thought and thought. What does every group have in common?. They thought about groups of friends, groups of flowers, groups of books, groups of houses, groups of clouds. They all seemed so different. What do they all have?

“Think in a different way,” said Zero. “Don’t think about what is in the group. Just think about the group itself.”

“Well, every group is a group of something,” suggested Rita.

“Right,” said Toby. “Every group has something in it.”

“You’re on the right track,” said Zero. We can call those things ‘members’ of the group.”

“So every group has members,” said Toby. “That’s what every group has in common.”

“Good,” said Zero. “But we can do better. Think about the members themselves.”

“I’ve got it!” exclaimed Rita! “Every group has a certain number of members.”

“Exactly!” said Zero. “Excellent!”

“Huh?” asked Toby. He thought it through out loud. “Some groups have five members, some have ten, others maybe 50 or 100. But sure enough, every group has a number of members.”

“Now,” said Zero, “one way to describe a group or to know about a group is by the number of its members.”

“Like if I say I have 10 cookies on a plate, that’s much better than having only one or two,” suggested Rita.

“If I have 12 books in my bag, I know it’s going to be very heavy,” said Toby.

“That’s right,” said Zero. “When we know the number of members of a group, it gives us a better understanding

of the group. It gives us more exact knowledge. One of the reasons we make math is to give us more exact knowledge.”

“So, we make groups, and each group has a number,” said Toby. “But where did numbers come from? Oh, no, don’t tell me. We make them, right?”

Zero just smiled.

4. Creating Numbers by Counting



“So where do numbers come from, Zero?” asked Toby again. “You say we make them, but how?”

“How did you first learn to count, Toby?” asked Zero.

“When I was a baby, I counted on my fingers,” said Toby.

“Very good,” said Zero. “How did you do that?”

“Like this,” broke in Rita. “He would look at a group of something, like apples in a bowl. Then for each apple, he would touch one finger.”

“That’s right,” said Zero. “What you do when you count on your fingers is you let each finger stand for a different member of the group. Actually, you create a group of fingers with the exact number of members as the group of apples.”

“I don’t get it,” said Toby. “I always had my fingers. I don’t create them when I count.”

“No,” said Zero. “I didn’t say you create your fingers. You make a group of fingers. You hold your fingers together in a group.”

“So where do numbers come in?” asked Rita.

“A number is an idea—an idea that you create. You make math, remember? Now, we have already learned that every group has a certain number of members. But who makes the groups? You do! As soon as you make a group, it automatically has a number—a number of members. The action of counting makes numbers. As I told Toby just now, in math, actions can make objects.”

“Let me see,” said Rita. “We create groups, and each group has members, a certain number of members. So when we count on our fingers, we create a group with our hands that has the same number of members as the group of apples we are counting, right?”

“That’s right,” said Zero. “Now then, each amount of fingers is different, right?”

“Sure,” said Rita.

“So,” said Zero, “we give each amount of fingers a different name.”

“I know the names!” shouted Toby. “One, two, three, four, five, six, seven, eight, nine, and ten!”

“That’s it,” said Zero.

“But now we are older,” said Toby. “We don’t need to count on our fingers.”

“That’s right,” said Rita, “and numbers go much higher than ten. How do we count without our fingers, and go higher than ten?”

“What you do now is skip the fingers part and just use the numbers,” said Zero. “You do it so naturally that you’re probably not even aware of what’s happening.”

“What do you mean?” asked Toby.

“You might not believe it,” said Zero, “but here is what happens in your mind when you count a group of apples. First you look at one apple, and in your mind you make a special group with one member, and call it ‘one’. Then you look at another apple, and in your mind you make a new group and call it ‘two’. Then with another apple, you make another group and call it ‘three’. With each apple you count, you create mental group and give it a new name. As you count, you create numbers.”

“That sounds like a lot of work,” said Toby doubtfully. “I thought you said this was going to be easy and fun.”

“The wonderful thing,” said Zero, “is that it is easy and fun. Once you’ve created these mental groups—the numbers—they now exist as objects, separate from apples and fingers. You can now use them to count anything!”

“Cool!” exclaimed Toby and Rita together.

“Your mind does this all so automatically that you don’t even notice. You just say ‘one, two, three’ and so on. When you’re done, you know how many apples you have. It’s so easy and natural that you aren’t even aware of how it’s happening.”

“That’s for sure!” Toby and Rita laughed.

5. Groups of Groups: Counting Past 10



“Hi, Rita, hi Toby. What are you doing?” It was Rita’s little sister, Amy.

“Hi Amy,” said Rita. “We are talking about counting.”

“Zero is teaching us,” said Toby.

“Who is Zero?” asked Amy.

“Here.” Toby held out his hand.

“Hi Amy,” said Zero.

“Hi,” said Amy, quietly, moving back a step or two. She was a little bit shy.

“Do you want to hold him?” asked Toby.

“Uh, OK,” said Amy. Toby put Zero gently into Amy’s hand. “Oh, he’s really nice,” she said.

“Hrmph,” coughed Zero. You could see he didn’t like being treated like a toy or a pet. “We were talking about counting.”

“Can you count to 100?” asked Amy.

“Sure,” said Zero. “That’s easy. 1, 2, 3, 4, ...” Toby and Rita joined in, “... 8, 9, 10, 11, 12 ...” they continued. “... 22, 23, 24, 25 ...” they went on, “... 32, 33, 34, 35 ...”

“Hey, wait a minute,” said Amy. “That’s not fair. You’re not going to 100. You’re just saying the same numbers over and over again.”

Toby and Rita laughed. “No, silly,” said Rita. “That’s the way you do it. You first count to 10, then to 20, then to 30, and so on all the way to 100.”

“Yeah, that’s right,” said Toby.

“Very good,” observed Zero. “But now, can you tell me why we do it that way?”

“Because that’s how we learned to do it in school,” said Toby.

“Not a good enough answer,” said Zero. “You make math, remember? Why do you suppose it is that way? Think.”

“Well, Amy said ‘it’s not fair’,” thought Rita out loud, “as if we were cheating or something.”

“I don’t know if we were cheating,” said Toby, “but it sure makes it easy. I like easy things. You only have to remember 10 numbers. Then you just repeat them.”

“That’s the idea,” said Zero, smiling. “But there is a little trick, too. You don’t just repeat one to ten. What else do you do when you count to 100?”

“Each time you count from one to ten you start with a different number, like 11 or 21 or 31 or 41 or 51, and so on,” said Rita.

“Now you’re talking,” said Zero. “So what is the trick?”

Toby and Rita thought for a minute. Amy looked at Zero, admiring his golden circular shape.

“I’ll give you a hint,” said Zero. “It has to do with groups.”

“Oh,” said Rita, “like groups of 10?”

“Very good,” said Zero. “That’s the trick. You make groups of ten, and then you count those groups.”

“But ten of what?” asked Toby.

“Ten of whatever you are counting,” said Zero. “In your mind, when you get to ten, you make a group, and the group becomes a single object. You count that as 1, and start again. That’s why we write ten as ‘10’. It means one group of ten, and nothing more. Eleven is one group of ten and one more, so we write it like this: ‘11’. When we get to twenty, that’s a second object like ten and nothing more, so we call that ‘20’. And on it goes.”

“Oh, oh, oh!” exclaimed Rita. “I just had an idea. When we were talking about counting on our fingers, I asked how we can count past ten. Maybe when people first invented numbers, they went up to ten on their fingers, then started over again.”

“Good one,” said Toby.

“Now you’re getting the hang of it,” said Zero. “You make math, but you are not the first people to do it. People have been making math for a long time.”

6. Nothing and Zero



“Is anybody hungry?” Toby’s mother came outside with a big bowl of peanuts. The children laughed with delight, and swarmed around her. They loved cracking open the peanut shells, finding the two or three peanuts packed inside, and popping them into their mouths.

Soon the bowl was empty and their tummies were full. Toby picked up two empty half shells and put them together. “Hey, Rita,” he said. “Here’s one more peanut for you.”

“Thanks, Toby,” said Rita. But when she took it, the shells fell apart, empty.

“Ha, ha, ha,” laughed the children together. “That was good joke, Toby,” said Rita. The children lay back on the grass, looking at the sky, daydreaming.

This counting past ten thing was something Toby liked. It was so easy. “All you have to know is nine numbers,” he thought, “1, 2, 3, 4, 5, 6, 7, 8, and 9.”

“Whoops!” said Zero. You forgot a very important number.

“What are you saying, Zero?” asked Rita, sitting up.

“Toby was thinking about how easy math is, because there are only nine numbers,” explained Zero. “But I told him he forgot a very important number.”

“But you just said when we get to ten we start over,” complained Toby. “Ten is just one and zero, right?”

“Ahem,” said Zero. “One and what?”

“One and zero. But zero isn’t a number, is it? Zero is nothing,” said Toby.

“Am I nothing?” asked Zero.

“Oh, sorry, Zero,” said Toby. “Of course not. I wasn’t talking about you. I was talking about zero in math.”

“Zero is a very important idea in math,” said Zero. “Believe it or not, it took a while before people understood how important it is.”

“Why do you say that, Zero?” asked Rita.

“Let’s go back to groups for a minute,” said Zero. “Remember a group is a collection of things. We call those things members of a group. Now, can you imagine a group with no members?”

“That’s impossible,” blurted Toby. “How can you have a group with no members? What kind of a group is that?”

“Wait a minute,” said Rita. “Maybe it is possible. What about that peanut shell you gave me just now? That was a group with no members, wasn’t it?”

“Well, yes,” said Toby. “I guess so.”

“If you use your imagination you can think of lots of groups with no members,” said Zero. “For example, what about the group of pigs that can fly.”

“Pigs can’t fly,” laughed Amy.

“That’s right,” said Zero. “So that is a group with no members.”

“I get it,” said Rita. “Like the group of my brothers. I don’t have any brothers, so there are no members in that group.”

“Sure,” said Zero. “You have zero brothers.”

“And I have zero sisters,” said Toby. “The group of my sisters has no members.”

“You’ve got the idea now,” said Zero. “When we count, we use zero, and the symbol ‘0’, to show that there are no members in a group.”

“So ‘10’ means that we have one group of ten with nothing extra, right?” said Rita.

“That’s the idea,” said Zero.

“So ‘70’ means we have seven groups of ten and nothing extra,” said Toby.

“Now you’re catching on,” said Zero.

“Hey, this isn’t so difficult,” said Toby. “I do make math.”

7. Adding: Joining Groups



“Well,” said Rita, “This zero idea is interesting, and I’m having fun thinking about math, but look at the mess we’ve made.” In their haste to eat the peanuts, the children had dropped all of the peanut shells on the ground.

“I have an idea,” said Rita. Let’s see who can pick up the most peanut shells.”

“Great idea,” said Toby.

Laughing and shouting, the children raced each other to collect the peanut shells and put them into little piles.

When they were all collected, they each counted the shells in their pile.

“I have 17,” said Rita.

“I’ve got 13,” said Toby.

“And I have 8,” announced Amy.

“Let’s count them all now to see how many we have,” suggested Amy.

“I don’t want to count them all again,” complained Toby.

“I do,” said Amy. “I want to know how many shells we have all together.”

“We don’t have to count them all again, Amy,” said Rita. “We just add the numbers together.”

“That’s right,” said Toby.

“Let’s see,” said Rita, thinking aloud. “What is 17 plus 13 plus 8? Hmm. 17 plus 13 is 30. Then we add 8 and it comes to 38.”

“Very good,” said Zero. “You have just demonstrated one of the most profound concepts of mathematics.”

“What does ‘demonstrate a profound concept’ mean, Zero?” asked Amy.

“It means to put an important idea into action, Amy,” explained Rita.

“Oh.”

“So, what profound concept have we demonstrated, Zero?” asked Toby.

“The concept is joining groups,” said Zero.

“Huh?” asked Toby.

“Joining groups,” said Zero again.

“But we didn’t join any groups,” said Toby. “The peanut shells are still in their same piles. We didn’t put them together.”

“Maybe not on the ground,” replied Zero, “but you made a group in your mind.”

“Really?” asked Rita.

“Sure,” said Zero. “Here’s what happened. First you asked yourselves how many peanut shells there are altogether. Just by having this thought, you made a mental group of all the shells together.”

“I guess so,” said Toby.

“But,” Zero continued, “you didn’t know how many there were. Now then, we could have put all the peanut shells together in a big pile on the ground and counted them, right? But it’s easier to do it with your mind.”

“I get it,” said Rita excitedly, “when we add numbers, we join groups in our minds!”

“What?” asked Toby.

“We join groups in our mind. We’ve already counted the shells in separate piles, right? We have three numbers. All we have to do is put those numbers together and get one big number. That’s what happens when we add numbers.”

“This is more complicated than counting,” grumbled Toby.

“It sounds complicated when I explain it,” agreed Zero. “But you do it that way because it is easier.”

“Adding is easier than counting?” asked Toby. “How?”

“You tell me,” said Zero with a smile. “What is easier, counting all the peanut shells again, or adding them? How long did it take to count them the first time, and how long to add them?”

“That’s right,” said Rita. “Amy wanted to count them all again, but we knew it was easier to just add the numbers.”

“Now you’re making sense,” said Zero, “now you’re making math.”

8. Adding is Easy



“Hey Rita,” said Amy suddenly. Come and help me ride my bike.” She jumped up and ran towards her house. Rita and Toby followed her. When they got to Amy and Rita’s house, Amy was sitting on her bike, waiting.

Rita held the back of the seat as Amy pedaled up and down the walk in front of the house. The bicycle wobbled and weaved, and Amy shrieked with delight at every turn. As Rita struggled to keep the bike upright, she wasn’t watching where Amy was steering.

“Watch out,” said Toby. “You’re going to hit that tree!”

But it was too late. Bang! Amy bounced out of the seat onto the ground. She wasn't hurt, though. She just sat on the ground, still laughing.

"When are you going to learn to ride all by yourself, Amy?" asked Toby.

"It's hard to ride a bike," said Amy. "But I want to learn. I like to go fast."

"Learning is like that," came a familiar voice.

"Hey, Zero," said Toby. In all the excitement, he had forgotten about his little friend. "Like what?"

"Learning can be difficult sometimes, but it makes things easy."

"What do you mean, Zero?" asked Rita.

"Remember just now we were talking about adding and counting? We all agreed that it is easier to add than to count. That's true for you, Rita, and for Toby. But what about Amy? She doesn't know how to add yet. She will have to learn."

"It wasn't easy to learn," said Toby. "I still make mistakes."

"That's true," said Zero. "But would you like to forget about adding and just count things every time you want to know how many you have?"

"No way," laughed Toby.

"You see," continued Zero, "math is all about making difficult things easy. It's like riding a bike. If you wanted

to walk across town, you could, but it would take an hour, and you would get tired. But on a bicycle, you can do it quickly and easily.”

“How is math like that?” asked Rita.

“With math you can do difficult things quickly and easily. Look, if you’re in a hurry to get across town, you need two things. First, you need a bike, right?”

“Sure,” said Toby.

“Next, you need to know how to ride it. Right, Amy?”

“That’s right, Zero,” piped Amy.

“Math is the same way—but it’s all in your mind. First you have some ideas, like groups and numbers. Then you learn how to work with those ideas. You learn how to join groups or add numbers together. It takes a little time at first, but when you get good at math you can do some pretty amazing things.”

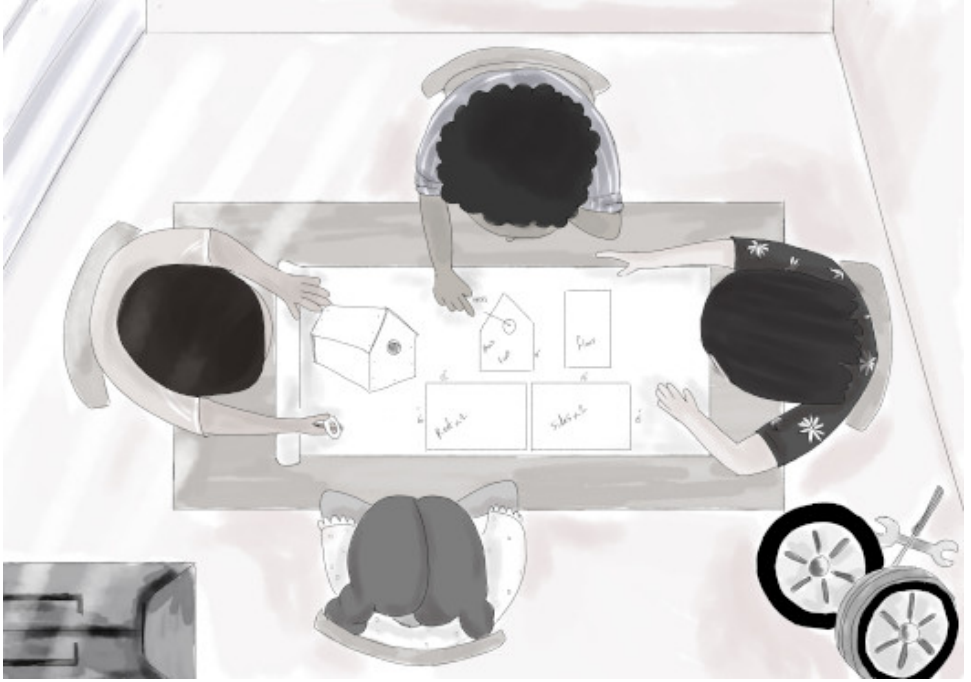
“Really?” asked Toby. “Like what?”

“Like build a bridge, travel around the world, cook a delicious meal, design a dress, discover a new planet, or cure a disease.”

“Wow”, cried Toby. “I can do all that with math?”

“And much more,” said Zero. “Math makes all of that easy.”

9. Symbols help us think



Amy pushed her bike back to the house. Rita and Toby followed, carrying Zero. They went into the garage to put the bike away. Rita's father, Mr. Kline, was sitting at the workbench, drawing.

"Hi, Dad," said Amy. "What are you doing?"

"I'm building a birdhouse, Amy," said Mr. Kline.

"Where?" asked Amy, looking up on the workbench. "I don't see any birdhouse. There's just a picture here."

"This is the plan," he said. "First I draw the plan, then I make the birdhouse."

“Why do you draw a plan?” asked Amy.

“Because I have to know what the birdhouse will look like,” replied her father.

“Your father is putting his idea on paper, Amy,” said Zero.

“Who’s that?” asked Mr. Kline, turning around in the direction of the voice.

“Here,” said Toby, holding up his hand. “This is Zero.”

“Well, I’ll be,” said Mr. Kline. “A talking ring. Where did this come from?”

“I created him,” said Toby, “from my mind. His name is Zero.”

“He’s teaching us about math,” added Rita.

“A talking zero, eh?” laughed Mr. Kline. “You kids have great imaginations. Well, Zero, just now you told Amy that my plan puts the idea for a birdhouse on paper. That makes sense.”

“You see, Amy,” continued Zero, “your father starts with an idea of a beautiful birdhouse that he wants to make. To build the birdhouse, it’s much easier to see it. So he puts his thoughts onto paper.”

“That’s right,” agreed Mr. Kline. “When I can see a picture or a plan, it’s much easier to build. Sometimes the ideas get all mixed up in my mind. Putting my ideas on paper helps me think more clearly.”

“Would you believe that math is the same way?” asked Zero.

“Sure,” broke in Toby. “Math is always getting mixed up in my mind!” Everybody laughed.

“Not just you, Toby,” said Zero. “Almost everyone. For example, we all know what ‘five’ is, but can you think of ‘five’ without thinking of the written symbol ‘5’? Or without thinking of five of something?”

“Wow, that’s hard, Zero,” said Rita.

“I can’t do it,” agreed Toby. “When I think of ‘5’ I see the number ‘5’.”

“I see five cookies,” said Amy.

“Even I can’t think of ‘five’ without thinking of the symbol ‘5’,” agreed Mr. Kline.

“But we all know what five is, right?” asked Zero. “We all agree that there are five nails here on the workbench. To put that idea out there where we can use it, we write it with the symbol ‘5’.”

“So numbers are just symbols,” mused Toby.

“Actually, Toby, numbers are ideas. Each number has a name, ‘one’, ‘two’, ‘three’ and so on. And each number has a symbol, ‘1’, ‘2’, ‘3’, and so on. The written symbols for numbers have a special name. We call them ‘numerals’.”

“Like Roman numerals?” asked Rita.

“Yes,” replied Zero. “The Romans used different numerals than we use today. Like ‘I’, ‘II’, ‘III’, and so on.

But they were symbols for the same numbers we use. Even very far back in history, people used symbols for numbers, to make it easy ...”

“... to make math.” Toby completed Zero’s sentence with a knowing smile.

10. Operators: the Tools of Math



The plan was finished, and Mr. Klein was ready to build the birdhouse. “Can we help you?” the children asked. They worked together. Mr. Kline showed the children how to saw pieces of wood and nail them together to form the floor, walls, and roof.

“Gee, dad, that’s really beautiful,” said Rita when it was done. “Can we hang it in the tree in the back yard?”

“Sure,” said Mr. Kline. The children took the birdhouse out back, and hung it from a low branch of a tree.

Then they waited to see if any birds would come to the birdhouse.

“I like building things,” said Toby. “We made such a nice birdhouse from a few pieces of wood.”

“How did you do it so quickly and easily?” asked Zero.

“We used tools,” said Rita. “We used a saw to cut the wood into pieces, and then a hammer to pound the nails.”

“Can you imagine trying to build a birdhouse without tools?” asked Toby. “We would have to break the wood with our hands.”

“And pound in the nails with our fists,” cried Rita. “That would be almost impossible!”

“It’s amazing what you can do with the right tools,” said Zero. “And did you know something? Math has—”

“No,” broke in Toby, “don’t say it. Don’t tell me now that math has tools, too.”

“But it does,” said Zero.

“Come on, Zero,” said Toby. “You must be joking. How does math have tools? There are no hammers, saws, or pliers in my desk at school.”

The children laughed.

“Not that kind of tool, fool,” said Zero with a wink. “I’m talking about tools that help you think. Do you know what those might be?”

The children were silent for a minute, thinking. “Hmmm ...,” said Rita at last. “You said that adding makes counting easy. Is adding a tool?”

“Yes,” said Zero. “Adding is a tool for joining groups together. Can you think of any other tools?”

“We used a saw to cut wood into pieces,” said Toby thoughtfully. “Maybe there’s a tool in math for separating groups.”

“Subtraction!” cried Rita. “That’s how we separate groups.”

“Very good,” said Zero. “Can you think of any more?”

“How about multiplication and division?” asked Rita.

“Excellent,” said Zero. “We’ll talk about them later. Now we know that math has tools. Because they are special mathematical tools, they have a special name. We call them ‘operators’.”

“Operators,” said Toby. “Wow.”

“Can you do operations with them?” asked Rita, giggling.

“Yes,” said Zero. “Adding, subtracting, multiplying, and dividing are all mathematical operations.”

“So we use mathematical operators to perform mathematical operations!” said Toby triumphantly.

“Isn’t that the same as saying we use the tools of math to work with groups?” asked Rita.

“Now you’re getting it,” said Zero

“So far we know how to join groups,” continued Rita, “and you said that subtraction means separating groups. Can you tell us more about that?”

“I’d be glad to,” answered Zero.

11. Subtracting: Separating Groups



“Hey children” said Mr. Kline, walking out of the garage with a ladder and a basket in his hands. “Can you help me pick some apples?”

“Sure,” they cried.

Mr. Kline set up the ladder, and Rita and Amy took turns climbing up and picking the apples, dropping them into the open hands of the others below. Toby put the apples into the basket, counting them for fun. “... 13, 14, 15, 16, 17 apples,” he called out when they were finished.

“That’s more than enough for an apple pie,” said Mrs. Kline, who had come out to watch. She looked into the basket. “Oh, what’s that?” she said suddenly, picking up an apple.

“Oh, no,” said Mr. Kline. “A worm got into that one. Are there any more? Children, could you please help us look?”

The children took the apples out of the basket one by one, and they all inspected them for worms. Soon they had five bad apples on the ground.

“Do we still have enough for a pie?” asked Toby, hopefully.

“Oh, yes,” said Mrs. Kline. “There are plenty. I can make two pies with 12 apples. Here, dear,” she said to Mr. Kline, “help me carry these into the kitchen.”

“How did she know that?” asked Amy.

“Know what?” said Toby.

“How did mom know there were 12 good apples left in the basket? She didn’t count them.”

“That’s easy,” said Zero. “Right, Toby?”

“Right,” said Toby. “It is easy. She just subtracted the number of bad apples from the number of all the apples.”

“Like Zero said,” joined in Rita. “Subtraction is how we separate groups. We had a group of 17 apples, then we took out 5 of them. $17 \text{ minus } 5 \text{ equals } 12$.”

“Oh,” said Amy.

“We separated the apples with our hands,” said Rita, thinking out loud. “Then we separated the groups in our mind, by subtracting the numbers. Is that right, Zero?”

“Yes, Rita,” said Zero. “Very good. You see, very often the math you make in your mind is directly related to something in the physical world.”

“What is the physical world?” asked Amy.

“It’s everything you see around you,” replied Zero. “The grass, the trees, the apples, the sun, the sky, your house, even your own body.”

“What other world is there?” asked Amy.

“Another world is the mental world, the world of your mind. You can imagine two suns, right?”

“Ha, ha, ha,” laughed Amy. “That’s funny, Zero.”

“Sure,” said Toby. “I can imagine two suns. But there is really only one sun.”

“In the physical world there is only one sun,” agreed Zero. “But in the world of your mind, there can be two. You can create all kinds of things in your mental world.”

“Wow,” said Amy. “That’s great.”

“Of course,” continued Zero, “you should be careful what you imagine. Now then, numbers exist in the mental world. We can use numbers to represent groups in the physical world.”

“What is ‘represent’?” asked Amy.

“Represent means to stand for,” said Zero. “Every group in the physical world is represented by a number in the mental world. A number in your mind stands for all the apples in the basket. When you subtract, you separate that group in your mind.”

“So,” said Toby, “We make math in the mental world, and we can use it to represent the physical world.”

“You got it,” said Zero.

12. Multiplying: Counting Groups



“Toby!” called Toby’s mother. “Come inside now and help me get ready for dinner.”

“OK, Mother,” said Toby. He said goodbye to Rita and Amy, put Zero in his pocket, and went into the house.

“Here, Toby,” said his mother. “Take these plates, glasses, knives, forks, and spoons and set the table. Your Uncle Otis, Aunt Sarah, and your cousins are coming for dinner. So, with three of us and four of them that makes ___”

“Seven,” said Toby. “There are two groups, our family and Uncle Ray’s family. We join those groups in our mind by adding, and get seven people. That’s how I know to put seven knives, seven forks, and seven spoons on the table.”

Toby’s mother looked at him with a puzzled expression. “That’s right, Toby,” she said. “You’re getting very mathematical all of the sudden.”

“I have a new friend, Zero, who is teaching me about math,” said Toby.

“Wonderful,” said his mother. “I hope he can help you improve your grades at school.”

Toby put seven plates and seven glasses on the table. Then he gathered up the silverware and laid out the knives, forks, and spoons at each place.

“Thank you, Toby,” said his mother. “Now, could you put away your school books and tidy up your room?”

“Sure,” said Toby.

As he was walking to his room, Zero asked him, “So Toby, I have a question for you. How many pieces of silverware did you put on the table just now?”

“OK, Zero,” said Toby, “I’m ready for you this time. I put seven knives, seven forks, and seven spoons on the table. To join them all together in one group, I add 7 plus 7 plus 7. Let’s see now—”

“Well, you could do it that way,” interrupted Zero. “But there is a faster, easier way to find out.”

“Faster and easier?” asked Toby. “I like fast and easy. What is it?”

“Count the groups,” replied Zero. “How many groups of silverware did you put on the table?”

“Knives, forks, and spoons. Three groups,” replied Toby.

“Each group had the same number of things in it, right?”

“That’s right. Each group had seven.”

“So you had three groups, and each group has seven members. What can you do?”

“I guess I could multiply,” said Toby. “3 times 7 is 21. But you said there was a faster and easier way than adding. Multiplying is harder than adding.”

“How so?” asked Zero. “Multiplying is just like counting. What could be easier than counting?”

“Counting?” asked Toby.

“Counting groups,” replied Zero.

“But for multiplication we have to learn all those times tables and stuff,” said Toby. “That’s hard.”

“Like learning to ride a bike?” asked Zero with a smile.

“OK, you got me there,” agreed Toby. “Once I learn them, things get faster and easier.”

“Try this,” said Zero. “How many things in total did you put on the table just now? I mean silverware and dishes.”

“Let’s see,” said Toby. “Knives, forks, spoons, plates, and glasses. Five groups of seven. 5 times 7 is 35.”

“Very good,” said Zero. “Now, do you want to go back to the table and count them all?”

“No way, Zero,” said Toby. “Multiplying was easier and faster. I just count the groups, then multiply.”

13. Multiplying: Groups of Groups



Toby put Zero on his desk and got to work tidying up his room. He put his school books away, straightened up his bed, and hung up his clothes. Then he began clearing his desk.

“Hey, Toby,” said Zero. “What are those?”

“Those are packs of chewing gum,” said Toby.

“Why are they two different colors?” asked Zero.

“The red ones are Chompies and the yellow ones are Golden Glow,” said Toby.

“Which do you like better?”

“I like Chompies better,” said Toby. “They are more fun to chew. That’s why I have more of them. Look, I have 5 packs of Chompies, and only 3 packs of Golden Glow.”

“Hmm,” said Zero, “it looks like the Golden Glow packs are bigger.”

“They are,” replied Toby. “They have five pieces in a pack. Chompies only have three pieces per pack.”

“Hold on, Toby. That means you have the same amount of Chompies as Golden Glow.”

“What?” asked Toby. “But I have 5 packs of Chompies, and only 3 packs of ... Wait a minute, Zero. Are you getting me to do math again?”

“What else?” said Zero with a smile.

“OK,” said Toby. “Let’s see. I have 5 packs of Chompies, with 3 in each pack. That’s 5 groups with 3 in each group. 5 times 3 equals 15. So I have 15 pieces of Chompies.”

“Good,” said Zero.

“Now, the Golden Glow. I have 3 packs of Golden Glow, with 5 in each pack. 3 times 5 equals 15. Hey Zero, you’re right! I have 15 pieces of Golden Glow, too. I have the same number of each.”

“And you showed me a couple of things about multiplication, too,” said Zero.

“How so?” asked Toby.

“The first thing you showed me is that when you multiply, you make groups of groups.”

“Groups of groups?” asked Toby.

“Yes,” said Zero. “Each pack of gum is a group, right?”

“Right,” said Toby. “Chompies come in groups of 5 and Golden Glow comes in groups of 3.”

“Then when you say you have 5 packs of Chompies, what you are saying is—”

“That I have a group of Chompies packs. I have a group of groups!” cried Toby. “And the packs of Golden Glow are another group of groups.”

Zero smiled. “Very good. Now, what did you learn about these groups of groups? How much gum do you have?”

“15 Chompies and 15 Golden Glow,” replied Toby. “Each group of groups has the same number of members.”

“Even though—” began Zero.

“Even though one is 5 groups of 3 and the other is 3 groups of 5. Wow. Is that true all the time, Zero?”

“Try it,” said Zero. “Look at your bed.”

Toby looked at the bed. The bedspread had a large checkerboard pattern of blue and white squares in the middle.

“How many squares are there?” asked Zero. “Count the rows.”

“Rows?” asked Toby. “Why rows?”

“Why do you think?” asked Zero.

“Hmm,” said Toby, thinking out loud. “Rows, rows ... each row has squares in it, so each row is like a group.”

“That’s right, Toby,” said Zero.

“And there are rows going across and rows going up and down. The ones going across each have four squares, and the up-and-down ones each have five squares. So some groups have four squares and some have five.”

“Can you group the groups?” asked Zero.

“Well, let’s see ... they could be 4 groups with 5 squares in each group, or 5 groups with 4 squares in each group,” said Toby.

“Which gives a total of—”

“20!” cried Toby. “Either way, it’s 20. 4 groups of 5 are the same as 5 groups of 4! Hey, Zero, this is cool.”

“And you did it all yourself,” said Zero.

14. Subtracting: Comparing Groups



“Toby! Come to the door! Lori and Ray are here!” Toby’s mother was calling him from the living room.

Toby rushed to the door. “Hi Ray, hi Lori,” he greeted his cousins.

“Hey Toby,” said Ray. “We’ve got some new Dundle cards.”

“Wow. Let me see,” said Toby.

“Look,” said Lori, pulling a bundle of cards out of her pocket. Each Dundle card had a picture of a cartoon character, with information printed on the back. “Here’s

Repro, Zantar, and Farjas. And this is Torad, a totally new Dundle.”

“Excellent,” said Toby. “I hope I can get one of those. Do you want to see my collection? Let’s go to my room.”

The children hurried to Toby’s room, laughing and talking excitedly about their card collections.

“Here are my cards,” said Toby, holding up a stack of cards.

“Wow,” said Ray. “You’ve got a lot. How many do you have?”

“39,” said Toby proudly.

“I have 30,” said Lori.

“And I have 25,” said Ray.

“You have more, Toby,” said Lori.

“How many more?” asked Ray.

“Why don’t we subtract and find out?” asked Lori.

“Good idea,” came a familiar voice.

“Huh?,” said Ray and Lori together. “Who said that?”

“Oh,” said Toby, “That was Zero.” He reached into his pocket and pulled out Zero.

“Cool, Toby,” said Lori. “Where did you get it?”

“He created me,” said Zero.

“Amazing,” said Ray. “He talks.”

“He’s been teaching me about math,” said Toby. “He says that I make math, and that everything about math comes from our minds. Can you believe it?”

“Sounds interesting,” said Lori. “What does it mean?”

“Let me give you an example,” said Zero. “Just now you three counted your cards and found out that Toby has the most cards. Then Ray wanted to know how many more cards Toby has. That question sparked a new thought in your mind—‘subtract.’”

“Hmm, interesting,” said Lori.

“Now, Lori,” said Zero, “Why did you suggest subtracting?”

“Well,” she said, “if Toby has 39 cards and I have 30, we have to find out the difference between them. So we subtract.”

“But subtraction means taking away,” objected Toby. “I didn’t take away any of my cards. I still have 39.”

“Lori is right, though,” said Zero. “Here’s how it happens. When you compare two numbers, what you really do is match the members of the two groups, one by one. So in her mind Lori matches her 30 cards with 30 of your cards.”

“I get it,” said Ray excitedly. “After those 30 cards are matched, Toby has 9 cards left over. The difference is 9. Toby has 9 more cards than Lori.”

“That’s it!” said Zero.

“But I didn’t match anything in my mind,” protested Lori.

“You may not be aware of it, but you did,” said Zero.
“How did you subtract?”

“I just said 39 minus 30 equals 9,” said Lori.

“Where did you get the 30?”

“That’s the number of my cards,” said Lori.

“True,” said Zero. “Now, did you take your cards from Toby?”

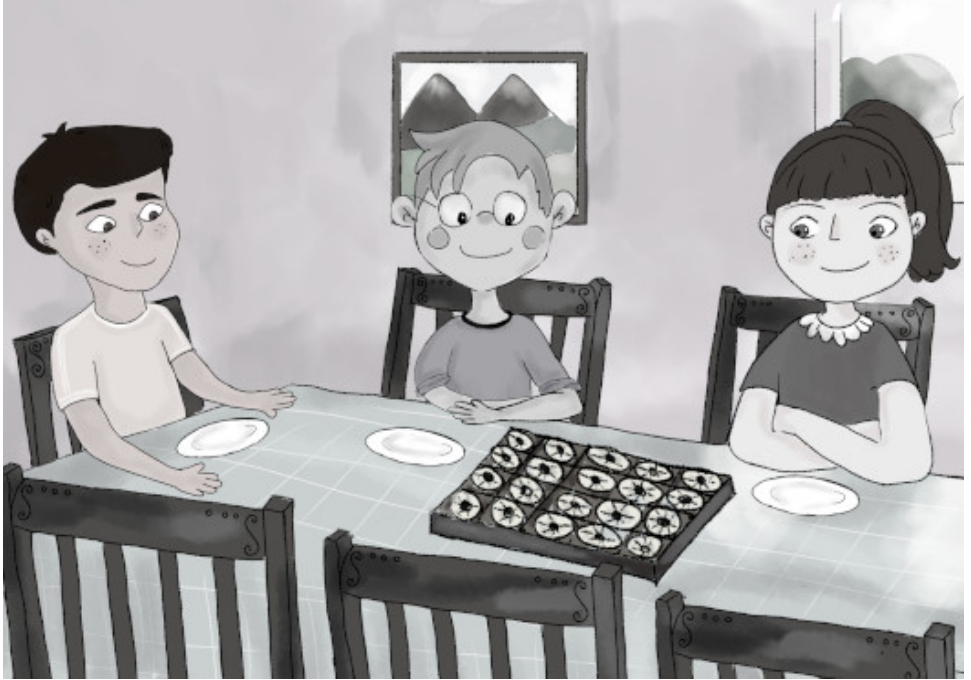
“No,” said Lori. “Of course not.”

“So how did you get ’39 minus 30’?”

“Oh, I see. The 30 is for 30 of Toby’s cards that match my 30 cards. Hey, this is really neat!”

“We’re making math,” said Ray.

15. Division: Many from One



“Something sure smells good,” said Ray suddenly. The children rushed to the kitchen to see Toby’s mother taking a golden roast chicken out of the oven.

“You children go to the table,” she said. “Dinner is ready.”

The two families sat down to a delicious dinner of chicken, rice, peas, and salad. After much conversation and laughter, they cleared the plates away, and Toby’s mother brought out the dessert.

“Oh, pineapple upside-down cake!” cried Toby, Lori, and Ray together. “Our favorite!”

Toby’s father took a knife and began cutting the cake. The cake was rectangular, so he cut it into four rows lengthwise, and five rows across.

Toby looked at the cake cut into 20 pieces, with 4 pieces along the ends and 5 pieces along each side. It reminded him of something.”

“Hey,” he said suddenly. “It’s just like my bedspread.”

Everyone looked at Toby.

“See,” he said. “Dad cut the cake into squares, like on my bedspread. It’s not blue and white, but it has the same number of squares: 4 rows by 5 rows.”

“Hmm, how about that?” said Toby’s mother. “You are very observant today.”

The children ate their cake with great delight, and then went back to Toby’s room.

“Look, you guys,” said Toby. “See my bedspread. It’s the same as our cake.”

“Sure enough, Toby,” said Zero.

“Hi, Zero,” said Lori. “Are you going to tell us more about math?”

“Actually, I was hoping you would tell me something,” said Zero.

“About the cake and the bedspread? What can we tell you?” asked Lori. “They both have the same pattern, with 20 squares.”

“How did the cake get that pattern?” asked Zero.

“Toby’s father cut it that way,” said Ray.

“Why did he cut it?” asked Zero.

“Well, if he didn’t, then I would have eaten the whole thing myself,” said Ray. The children all laughed.

“He divided it up so we could each have a piece,” said Lori. “Oh, I get it—division. Zero is going to tell us that we make division, like we make math.”

Zero smiled.

“We make division,” repeated Ray, thoughtfully. “We divide things. Uncle Vern cut the cake into 20 equal pieces so that we could each have some. Division is something we do.”

“Excellent,” said Zero. “Now here’s a question for you. How does division relate to groups?”

“Groups?” asked Lori. “What do you mean, groups?”

“A group is collection of objects. It is a single thing with many members,” said Zero.

“Zero has been showing us that we make math by working with groups,” said Toby. “We group things together because it’s easier to work with one thing than with many things. Adding means joining groups together.

Subtraction is separating and comparing groups. When we multiply we make groups of groups.”

“So where does division fit in?” asked Lori.

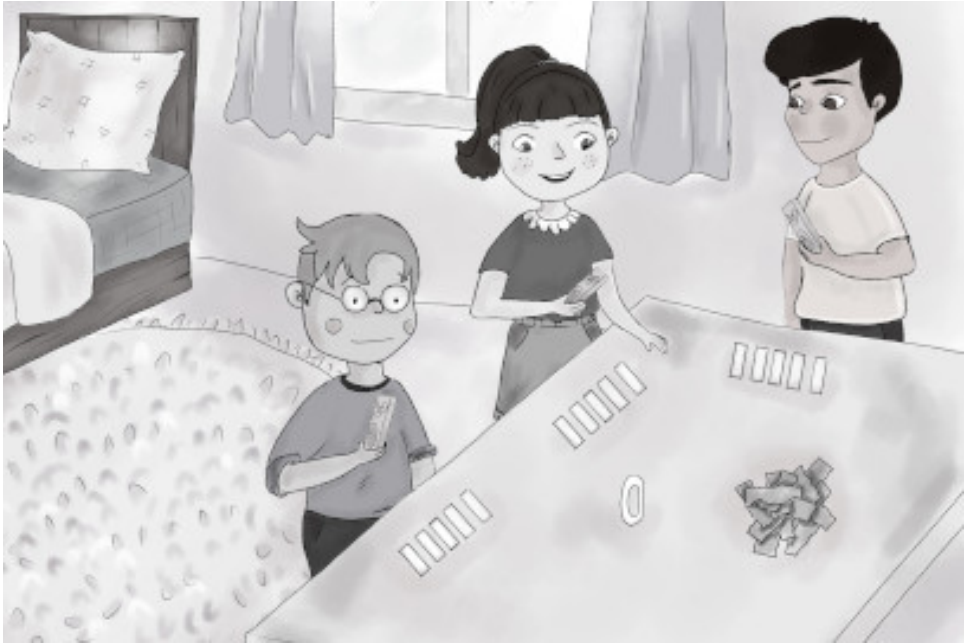
“When Toby’s father cut the cake, he divided it into pieces,” said Ray.

“That’s it!” exclaimed Toby. “He made a group out of a single thing. Division creates groups. It turns one into many.”

“Now you’re doing it,” said Zero. “Now you’re making math.”

“Amazing,” sighed Lori.

16. Division is Sharing



“Look here, Lori and Ray,” said Toby, “I have something for you.” He reached across to his desk and gathered up all of the packs of Golden Glow and Chompies. “Do you like these?”

“Of course, Toby,” said Lori.

“Wow, Golden Glow,” said Ray. “Are those for us?”

“There’s a pack here for each of us,” said Toby, handing one to Ray, one to Lori, and keeping one for himself.

“Thanks, Toby,” said Ray. Lori smiled with appreciation.

“You’re welcome,” said Toby. “And here, I want to share my Chompies with you too.”

“That’s very generous of you, Toby,” said Lori. “Thank you.”

“Hmm,” said Toby. “There’s a problem. I have five packs of Chompies, but there are just three of us. I can’t share them equally.”

“That’s OK,” said Ray. “I’ll just take one pack, and you and Lori can each have two.”

“Thanks, Ray,” said Toby, but I really want to share these equally. There must be a better way.”

“Try opening the packs,” came the clear, quiet voice of Zero.

“Why Zero?” asked Toby.

“Try it and see.”

So Toby opened the packs of Chompies and put all the pieces into one pile.

“How many Chompies do you have?” asked Zero.

“I know,” said Lori. “15. There were five packs, and three in each pack. 5 times 3 equals 15.”

“So, can you share them equally now?” asked Zero.

“Of course,” cried Toby. “Divide 15 by 3. It’s so easy once you see it.”

“Good thinking, Toby,” said Zero.

Toby counted out 5 Chompies each for Ray, Lori, and himself.

“There,” he said. “Now they are equally divided.” The children each unwrapped a piece of gum and popped it into their mouths.

“I just realized something,” said Lori.

“What?” asked Toby.

Lori continued, “Remember how I counted the Chompies by multiplying: 3 times 5 equals 15? Then we divided the Chompies by saying 15 divided by 3 equals 5. Multiplying counts groups of groups. Dividing separates one group into many equal groups.”

“Hmm,” said Ray. “It seems they are similar somehow.”

“Look,” said Toby suddenly. “Look at the bedspread. Division is the reverse of multiplication.”

“What do you mean?” asked Ray.

“This afternoon Zero showed me multiplication using the bedspread. See how the checkerboard pattern is 4 squares by 5 squares? 4 times 5 equals 20. There are 20 squares.”

“Oh, I get it,” said Lori. “The cake was divided into 20 pieces. 20 divided by 5 equals 4.”

“Yes,” said Toby. “Or 20 divided by 4 equals 5.”

“That’s pretty neat,” said Ray. “I see how the numbers are the same. But Lori said multiplying counts groups of groups. Is division counting, too?”

The children all thought for a minute.

“Maybe it is,” said Toby. “Maybe division counts small groups that make up a big group. Yes, that’s it! 15 divided by 3 equals 5. We each got 5 pieces of gum. There are 3 small groups in 15, and each of them has 5 members.”

“So division lets us share equally by counting groups and members. Is that right, Zero?” said Lori. “Hey, Zero, you’re very quiet. Is everything OK?”

The children looked at Zero. “Sure,” he said, warmly. “Everything is great. I don’t have to say anything. You’re doing it on your own. You are making math.”

17. Equality



After Toby's cousins, aunt, and uncle went home, it was time for Toby to get ready for bed. He brushed his teeth and put on his pajamas. Sitting in bed, before turning out the light, he was looking at the checkerboard pattern on his bedspread when suddenly a thought came to him.

"There are 4 rows of 5 squares," he thought, "which makes 20 squares. But if you look at it another way, there are 10 squares on the left side, and 10 squares on the right side. 10 plus 10 equals 20."

"What are you thinking?" asked Zero.

“Look, Zero,” said Toby. “Isn’t it interesting that 4 times 5 equals 20, and 10 plus 10 equals 20.”

“Why is that interesting?”

“Well,” said Toby, “Does it mean that 4 times 5 equals 10 plus 10? After all, they both equal 20.”

“What do you think, Toby?”

“I think it does. Yes, 4 times 5 does equal 10 plus 10.”

“Does it also equal 25 minus 5?” asked Zero.

“Hmm, let me think about that. Let’s see ... 25 minus 5 also equals 20, so yes, it should equal 4 times 5, and 10 plus 10.”

“What about 40 divided by 2?”

“That’s 20, so yes, it must be equal to the others. That’s pretty cool, Zero.”

“What does this tell us about groups?, Toby?” asked Zero.

“It seems like you can get a group in many different ways. You can get to a certain number in many ways: by adding, multiplying, subtracting, or dividing. They are all ways to get to the same group,” said Toby.

“So what does that tell us about how we think?” asked Zero. “After all, you are making math.”

“They are all equal,” said Toby. “We see that they are all equal. But they are different, too. They are different ways of saying the same thing.”

“And so your mind—” started Zero.

“My mind sees the different ways, but it finds the equality,” said Toby.

“And finding equality helps you discover unity,” said Zero. “We often see things as different when really they are the same. It’s all in how you look at things.”

“You know, Zero, looking at you just now, it seems you are growing bigger,” said Toby.

“Who, me? Are you sure?”

“Pretty sure. Is that possible?”

“Maybe,” said Zero, “as your understanding of math grows, I grow too.”

Toby thought about that for a while. “Oh no!” he cried suddenly.

“What’s wrong?” asked Zero.

“I just remembered. I have that math test tomorrow. I didn’t study at all today!”

Zero started laughing.

“What’s so funny, Zero?” asked Toby.

“You said you didn’t study. What do you think we’ve been doing all day?”

“We weren’t studying,” said Toby. “We were having fun.”

“Did you learn anything?” asked Zero.

“Sure,” said Toby. “I learned a lot. I learned about groups, counting, adding, and all sorts of other things. And I learned that I make math.”

“Do you understand better now?”

“I guess I do,” said Toby. “I’m beginning to understand.”

“So what kind of studying could be better than that? Instead of just memorizing, now you know what you do when you make math.”

“But what about the test?” protested Toby.

“What about it?” replied Zero. “I’d say you’re ready for it. You understand that you make math. So go for it. Use your mind: your memory, your creativity, your imagination, and your logic. You can do it because you make math.”

“Well,” said Toby, “when you put it that way, it doesn’t sound so bad.”

“Go to sleep now, and see what tomorrow brings,” said Zero.

18. Toby and the Test



The next day dawned cool and clear. Toby woke up with a sinking feeling in his heart—the math test! But then he remembered. “Zero,” he said. “Are you still here?”

“Of course,” came a familiar voice. “I’m not leaving you. I’m part of you. I’m with you all the way.”

“Today is the math test,” Toby said. “Are you ready?”

“Me?” asked Zero. “I’m always ready. How about you?”

“I think so,” said Toby, “as long as you are here to help.”

Toby ate breakfast quickly, gathered his books, and headed for the door.

“You’re in quite a rush today,” his mother said. “Is something special happening at school?”

“Just a math test,” said Toby with a laugh—much to her surprise—and quickly closed the door behind him.

Outside he saw Rita coming out of her house. “Hey, Toby, wait up!” she called. The two of them walked together to school. “Good luck with the test,” she said when they got to school. “I’m sure Zero will come in really handy.”

Math was the last class of the morning, just before lunch. All through history and English class, Toby kept looking at the clock. He was a little scared and also excited at the same time. He had never cared so much about math before.

Finally the time came. The teacher started handing out the tests.

“Hey, Zero,” whispered Toby. “Are you ready?”

“Who are you talking to, Toby?” asked the teacher.

“Oh, uh, nobody,” said Toby.

“Shh,” said Zero. “You don’t need to say anything. I can hear your thoughts.”

“Wow,” thought Toby. “This is too much.”

“Don’t be too surprised,” came the reply. “After all, you made me with your mind.”

“Just like I’m going to make math in this test,” thought Toby. “Now, let me see ... this part about addition and

subtraction ... that's just joining and dividing groups—easy-peasy. And this section on Roman numerals, why those are just different symbols for the same numbers. Multiplication of big numbers ... oh I get it, first you multiply the ones, then the tens, then the hundreds. Group by group, step by step. It all makes sense now. Hey, Zero, you still there?"

"Sure," came the reply in his mind, "I'm listening. You are right on target. Keep it up."

"Equations are just different ways of saying the same thing," Toby thought. "And what about fractions? Zero, we didn't talk about those."

"Think it through," he heard. "You make math."

"OK, let's see ... one half is where you take a group with one member, then divide that group into two pieces, like the pineapple cake. So fractions are like division. Half of last night's cake is 10 pieces out of 20. $10/20$ equals $\frac{1}{2}$. It all makes sense."

His teacher noticed something different about Toby today. He wasn't staring out the window, tapping his pencil, or wrinkling his brow in frustration. He was focusing on the test, and even smiling occasionally. What had happened?

"Time's up, pencils down," she finally announced. The students got up, filed in line and handed in their tests on the way out the door.

“How did you do on the math test?” asked Rita in the hallway. “Did Zero help you?”

“Sure. Of course he did,” said Toby. “But he didn’t tell me the answers. He let me figure things out for myself. And I think I know why.”

“I do too,” exclaimed Rita. “Because you make math!”
They laughed all the way to lunch.



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